

Stability of the E2/M1 ratio at the T-matrix pole

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Abstract

We show how the stability of the E2/M1 ratio, evaluated at the T-matrix pole, can be understood given a much wider variation at the K-matrix pole.

Interest in the E2/M1 ratio has recently increased, due largely to new and precise pion photoproduction experiments spanning the first resonance region[1]. Several novel theoretical works have also attempted to relate this quantity to predictions given by quark models[2, 3]. While this ratio of electric quadrupole (E2) and magnetic dipole (M1) amplitudes is usually evaluated at the K-matrix pole, the Mainz group[4] has shown that it is particularly stable when evaluated at the T-matrix pole position. Models which gave K-matrix ratios differing by factors of two or three were shown to have very similar T-matrix ratios. This stability was subsequently verified by both the VPI and RPI groups[5, 6]. In the following, we show how these features can be understood within a simple parameterization scheme.

At the T-matrix pole, the E2 and M1 residues are complex, resulting in a complex ratio. This differs from the K-matrix ratio ($\text{Im}E_{1+}^{3/2}/\text{Im}M_{1+}^{3/2}$, at the point where $\text{Re}M_{1+}^{3/2} \rightarrow 0$) which gives a purely real number. These residues are determined either via the “speed plot” method[4] or by continuing the parameters of a model into the complex energy plane[5]. Quantitative results vary slightly, depending on details of the extrapolation procedure[7]. In the present study we have continued our fits to the pole. Qualitatively similar results should follow from speed plot determinations.

We have fitted the pion photoproduction database using multipoles of the form

$$M = \alpha(1 + iT_{\pi N}) + \beta T_{\pi N}, \quad (1)$$

where α includes the projected Born terms and both α and β contain kinematic factors required for correct threshold behavior. The πN T-matrix ($T_{\pi N}$) enters this parameterization in a direct way and provides most of the

required energy dependence. For energies of interest, the T-matrix is essentially elastic, and thus the first term is proportional to $\cos \delta e^{i\delta}$, δ being the associated πN phase shift. A term of this form arises naturally in many unitarization schemes, dynamical models, and solutions of the associated dispersion relations[8, 9, 10, 11]. The second term, proportional to $T_{\pi N}$ accounts for the energy dependence of the dominant M1 amplitude and gives a non-zero E2/M1 ratio at the K-matrix pole (the first term, being proportional to $\cos \delta$, vanishes at this point). Phenomenological polynomials in energy were included in both α and β in order to account for missing energy dependence. However, in fitting the restricted energy range (250–450 MeV) covering the delta resonance, it was found that only a constant factor was required for β . In addition, since the Born terms are slowly varying in this region, our extrapolations to the T-matrix pole required only a simple linear function of energy for α .

Results for the E2/M1 ratio, (at both the K- and T-matrix pole) using the above form, have closely matched those derived using either effective Lagrangian or dispersion-relation approaches, when similar databases have been fitted[12]. This suggests that our results should (at least qualitatively) agree with those found using other methods.

In Table I, we compare fits to two different databases[13] which result in very different E2/M1 ratios at the K-matrix pole. Values for the E2 and M1 residues at the T-matrix pole are also given. These have been split to show the contributions from the first and second terms in Eq. (1). This comparison is particularly revealing, as the first term vanishes at the K-matrix pole but does not vanish at the T-matrix pole. In fact, the term proportional to $(1 + iT_{\pi N})$ gives the dominant contribution to E2 at the T-matrix pole. As expected, the second term, proportional to $T_{\pi N}$, dominates M1. This immediately explains why the T-matrix pole ratio is so stable. The dominant contribution to E2 is controlled mainly by the Born terms, which are essentially fixed in any model[14]. It is also instructive to compare the variation of the β term at both the K- and T-matrix poles. In the two fits we have presented, the K-matrix pole ratio varies by a factor of five. The same level of variation is seen in β at the T-matrix pole. In fact, within this simple parameterization, the ratio β_{E2}/β_{M1} at the T-matrix pole is equivalent to the full E2/M1 ratio at the K-matrix pole.

From the above discussion it should be clear that the T-matrix pole ratio will remain finite even if the K-matrix ratio is zero. Thus, as has been

suggested previously[7], these two ratios probe different parts of the photo-production amplitude. Within our parameterization scheme, there is a simple relation between the K- and T-matrix ratios

$$E2/M1|_{\text{T-matrix}} \approx E2/M1|_{\text{K-matrix}} + i\alpha_{\text{E2}}/\beta_{\text{M1}}|_{W_{\text{pole}}}, \quad (2)$$

if one neglects $\alpha_{\text{M1}}/\beta_{\text{M1}}$, which from Table I is a good approximation. Since α and β are functions of energy, they become complex at the T-matrix pole (W_{pole}) and contribute to both the real and imaginary parts of the T-matrix ratio[15]. From Table I, we see that the term $i\alpha_{\text{E2}}/\beta_{\text{M1}}$ is about 0.05 in modulus, and is predominantly imaginary. This gives a rough way to see why the imaginary part of the T-matrix pole ratio remains more stable than the real part, as the K-matrix ratio goes to zero.

In summary, we have given a simple way to understand the relative features of the E2/M1 ratio at the K- and T-matrix poles. We hope this can be of some pedagogical value. The more difficult question of a separation into bare/dressed or resonant/background contributions must still be addressed if one wishes to compare with models.

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- [14] The contribution from Born terms times $\cos \delta e^{i\delta}$ can be compared to the phenomenological multipole amplitudes using the SAID program (Telnet: said.phys.vt.edu, userid: said).

- [15] There would be a sharper correspondence between the real part of the T-matrix pole ratio and the (real) K-matrix value if α and β were energy independent. See L. Resnick, Phys. Rev. D **2**, 1975 (1970) for an interesting and related study exploring the consequences of energy-independent Born terms in the Muskhelishvili-Omnès formalism.

Table 1: Comparison of K-matrix and T-matrix results for the E2/M1 ratio. Fits A and B result in E2/M1 ratios, at the K-matrix pole, of -1.93% and -0.39% respectively. At the T-matrix pole, Fits A and B have E2/M1 ratios of (modulus, phase) $(0.066, -127^\circ)$ and $(0.049, -100^\circ)$ respectively. Here α and β refer to the two terms in Eq. (1).

	E2		
	α -term	β -term	Total
Fit A	(1.12, -143°)	(0.42, 157°)	(1.38, -158°)
Fit B	(0.98, -126°)	(0.08, 157°)	(1.01, -131°)
	M1		
	α -term	β -term	Total
Fit A	(3.2, -136°)	(21.9, -23°)	(20.9, -31°)
Fit B	(3.2, -136°)	(21.7, -23°)	(20.7, -31°)